A KALMAN FILTER FOR TRACKING CONSTANT THRUST THEATER BALLISTIC MISSILES[1]

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ABSTRACT

Tracking Theater Ballistic Missiles (TBMs) during boost phase is becoming a requirement for some engagement strategies. The requirement for boost phase tracking can be based on the needs of an early launch doctrine or can be motivated by the availability of a cue provided by an airborne search and track infrared/laser system. The standard filters such as $\alpha-\beta$ or $\alpha-\beta-\gamma$, require that the target being tracked have, respectively, constant velocity or constant acceleration. If a TBM does not have constant acceleration during boost, an $\alpha-\beta$ or an $\alpha-\beta-\gamma$ filter may not provide valid estimates of the TBM's position and velocity and, as a result, may not be able to maintain track. TBM boost profiles assuming a constant thrust and a constant rate of fuel consumption show a hyperbolically increasing acceleration. This paper presents and analyses a Kalman filter designed to track a missile experiencing hyperbolically increasing acceleration during boost phase. For purposes of comparison, the tracking results of an $\alpha-\beta$ filter are provided for a general class of TBMs representative of the threat set.

I. Introduction

This paper presents a Kalman filter design capable of tracking a missile during boost phase. Boost phase tracking is becoming increasingly important based on requirements of an early launch doctrine, the availability of a timely non-organic cue, or the requirement for missile typing. Since the acceleration of a TBM in boost is rapidly increasing, from one g at launch to over $8g$ at burn-out, a standard $\alpha-\beta$ filter or $a-\beta-\gamma$ filter may not be applicable. Zarchan[1] discusses TBM boost phase velocity and acceleration characteristics. The two main assumptions in Zarchan are that the thrust and fuel consumption rate are constant. In this paper simulated TBM trajectories are examined to show that these assumptions are indeed valid. These assumptions are used in the filter design presented in this paper. Other assumptions used in the filter design are that during boost the thrust is much greater than gravity and the angle of attack is approximately zero.

The proposed filter has five states; position and velocity states for each dimension in the pitch plane and one state for total acceleration. Throughout this study it is assumed that the observables are limited to missile position. It will be shown that a TBM with constant thrust and fuel-consumption-rate has a hyperbolic acceleration profile. It is well known that an $\alpha-\beta$ filter has zero steady state estimation error when tracking a target with zero acceleration, constant error when tracking a target with constant acceleration and that the $\alpha-\beta$ filter diverges when tracking a target with ramp acceleration. Likewise, an $\alpha-\beta-\gamma$ filter diverges when tracking a target with parabolic acceleration. Hence, these filters are not appropriate for tracking a TBM in the boost phase. In this paper, the proposed full Kalman filter will be presented as well as the steady-state version.

[1]Approved for public release, distribution is unlimited

II. Derivation of Kalman Filter Dynamics

Let W_{TOT} be the total weight of the TBM at the launch, with the launch time being $t_0 = 0$. Let $\dot{W}$ denote the constant fuel-consumption-rate of the TBM (a negative constant). Then, the weight of the TBM at time t is

$$W = \dot{W} \cdot t + W_{TOT} \; .$$

From $F = ma$, we have

$$a = \frac{F}{m} \approx \frac{T}{\frac{\dot{W} \cdot t + W_{TOT}}{g}} = \frac{Tg}{\dot{W}} \cdot \left(\frac{1}{t + \frac{W_{TOT}}{\dot{W}}} \right)$$

$$= -S_I \cdot g \cdot \left(\frac{1}{t + S_W} \right) \; , \qquad (1)$$

where the specific impulse, S_I, and specific weight, S_W, are defined by

$$S_I = \frac{-T}{\dot{W}} \; ,$$

and

$$S_W = \frac{W_{TOT}}{\dot{W}} \; .$$

T is booster thrust - the dominant force by assumption. The constant thrust assumption implies that S_I is a positive constant and S_W is a negative constant. Both have units of seconds. From (1), the acceleration of the TBM is hyperbolic. Clearly, $|S_I| \geq |S_W|$.

(Evaluating (1) at $t = 0$ results in

$$a_0 = -\frac{-T}{\dot{W}} \cdot g \cdot \frac{\dot{W}}{W_{TOT}} = \frac{T}{W_{TOT}} \cdot g = \frac{T}{M_{TOT}} \; ,$$

which is the correct initial acceleration.)

Integrating (1) gives

$$v(t) = -S_I \cdot g \cdot \ln(-(t + S_W))$$
$$+ S_I \cdot g \cdot \ln(-S_W) + v_0 \; . \qquad (2)$$

Integrating (2) gives

$$p(t) = -S_I \cdot g \cdot ((t+S_W) \cdot \ln(-(t+S_W)) - (t+S_W))$$
$$+(S_I \cdot g \cdot \ln(-S_W) + v_0) \cdot t$$
$$+ S_I \cdot g \cdot (S_W \cdot \ln(-S_W) - S_W) + p_0 \; . \qquad (3)$$

From (1)

$$a \cdot (t + S_W) = -S_I \cdot g \; .$$

Differentiating,

$$\dot{a} \cdot (t + S_W) + a = 0 \; ; \qquad (4)$$

hence, from (1)

$$\dot{a} = \frac{-a}{(t + S_W)} = \frac{-a}{\frac{-S_I \cdot g}{a}} = \frac{a^2}{S_I \cdot g} \; . \qquad (5)$$

Also, substituting (1) into (4) results in

$$\dot{a} = \frac{S_I \cdot g}{(t + S_W)^2} = \frac{1}{S_I \cdot g} \left(\frac{S_I \cdot g}{t + S_W} \right)^2 \; . \qquad (6)$$

Equations (5) and (6) actually provide three different forms for $\dot{a}$. The time-invariant form given in (5) will be used as one of the filter plant state equations.

Next, specific weight and impulse are computed in terms of the TBM kinematic variables, p, v and a (and $\dot{a}$). From (4),

$$S_W = -\left(\frac{a}{\dot{a}} + t \right) \qquad (7)$$

and

$$S_I = -\frac{a \cdot (t + S_W)}{g} = \frac{a^2}{\dot{a} \cdot g} \; . \qquad (8)$$

Equations (7) and (8) may be applied to TBM data to validate the constant thrust assumption.

Filter Plant State Equations - One Dimensional Case

Filtering in one dimension is the simplest case, and will be used for comparison with the $\alpha - \beta$ filter. For motion in the TBM pitch plane, the one dimension is along the trajectory. The filter states are position, velocity and acceleration. The state equation is

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= a \\ \dot{a} &= \frac{a^2}{g \cdot S_I} \end{aligned} \; .$$

The state vector is $X = (x, v, a)'$. The measurement (output) equation is

$$z = x \ .$$

(The process and measurement noise terms do not appear in these equations. They will be included after discretizing and linearizing.) Discretizing gives

$$\begin{aligned} x_{k+1} &= x_k + \Delta t \cdot v_k \\ v_{k+1} &= v_k + \Delta t \cdot a_k \\ a_{k+1} &= a_k + \Delta t \cdot \frac{a_k^2}{g \cdot S_I} \ . \end{aligned}$$

We follow the formulas for the extended Kalman filter given in Anderson and Moore[2], section 8.2. The time update equations are

$$\left. \begin{aligned} \widehat{x}_{k+1|k} &= \widehat{x}_{k|k} + \Delta t \cdot \widehat{v}_{k|k} \\ \widehat{v}_{k+1|k} &= \widehat{v}_{k|k} + \Delta t \cdot \widehat{a}_{k|k} \\ \widehat{a}_{k+1|k} &= \widehat{a}_{k|k} + \Delta t \cdot \frac{\widehat{a}_{k|k}^2}{g \cdot S_I} \end{aligned} \right\} = f(X_{k|k}) \tag{9}$$

for the state, and for the covariance

$$\Sigma_{k+1|k} = F_k \Sigma_{k|k} F_k' + Q_k \ . \tag{10}$$

The matrices F_k and Q_k will be defined subsequently. The measurement update equations are

$$\widehat{X}_{k|k} = \widehat{X}_{k|k-1} + L_k[z_k - \widehat{x}_{k|k-1}] \tag{11}$$

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1} H_k [H_k' \Sigma_{k|k-1} H_k + r_k]^{-1} \cdot H_k' \Sigma_{k|k-1} \tag{12}$$

where

$$L_k = \Sigma_{k|k-1} H_k [H_k' \Sigma_{k|k-1} H_k + r_k]^{-1} \ , \tag{13}$$

(note that $'$ denotes transpose)

$$F_k = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & \left.\dfrac{\partial f_3}{\partial a}\right|_{X=\widehat{X}_{k|k}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & (1 + 2 \cdot \Delta t \cdot \dfrac{1}{g \cdot S_I} \cdot \widehat{a}_{k|k}) \end{bmatrix}$$

and

$$H_k' = [\ 1 \quad 0 \quad 0\ ] \ .$$

It should be noted that (H_k', F_k) is completely observable. The measurement noise term is r_k. It is dependent on the radar. The 3×3 matrix Q_k is derived from the process noise. Process noise for position and velocity could be set to zero. The process noise for the acceleration is set to account for the deviation of S_I from a constant value. In this case, the process noise is

$$p_k' = [\ 0 \quad 0 \quad \left(\frac{\partial f_3(X_{k|k})}{\partial \overline{S}_I}\right) \cdot \Delta S_I\ ] \cdot n_k$$

$$= [\ 0 \quad 0 \quad \left(\frac{\Delta t \cdot \widehat{a}_{MAX}^2)}{g \cdot \overline{S}_I^2}\right) \cdot \Delta S_I\ ] \cdot n_k \tag{14}$$

where the maximum acceleration $\widehat{a}_{MAX}$ is used to make the process noise and the state independent; $\{n_k\}$ is unit covariance white noise. The Q matrix is

$$Q_k = \mathrm{E}[p_k \cdot p_k']$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \left(\dfrac{\Delta t \cdot \widehat{a}_{MAX}^2)}{g \cdot \overline{S}_I^2}\right)^2 \Delta S_I^2 \end{bmatrix} .$$

Analysis of simulated TBM data suggests that some TBMs have a specific impulse, S_I, which deviates from a constant value more than other TBMs. ΔS_I would be chosen large enough to cover any TBM. The ultimate values for all of the noise terms would depend on the residual (there is only one since the output a scalar). The filter would give estimates for all three states. On line estimation of S_I is a topic for future study.

Filter Plant State Equations - Two Dimensional Case

In this section, the two dimensional case of the above filter is developed. The two dimensions are altitude, z, and down range, x, as defined by the pitch plane of the TBM. It is assumed during boost that the TBM angle of attack is zero. This assumption will simplify the two-dimensional formulation. The flight path angle is denoted by γ; hence,

$$\gamma = \tan^{-1}(\frac{v_z}{v_x})$$

where $v_x = \dot{x}$ and $v_z = \dot{z}$. From the assumption,

$$a_x = a_{TOT} \cdot \cos(\gamma) = a_{TOT} \cdot \frac{v_x}{\sqrt{v_x^2 + v_z^2}}$$
$$a_z = a_{TOT} \cdot \sin(\gamma) = a_{TOT} \cdot \frac{v_z}{\sqrt{v_x^2 + v_z^2}}$$

where a_{TOT} is the total acceleration. The state equations are

$$\begin{aligned} \dot{x} &= v_x \\ \dot{z} &= v_z \\ \dot{v}_x &= a_{TOT} \cdot \cos(\gamma) \\ \dot{v}_z &= a_{TOT} \cdot \sin(\gamma) \\ \dot{a}_T &= \frac{a_{TOT}^2}{g \cdot S_I} . \end{aligned}$$

The state vector is five dimensional in this case:

$$X = (x, z, v_x, v_x, a_{TOT})' .$$

Repeating the above steps, discretizing the continuous time state equations gives

$$\begin{aligned} x_{k+1} &= x_k + \Delta t \cdot v_{x,k} \\ z_{k+1} &= z_k + \Delta t \cdot v_{z,k} \\ v_{x,k+1} &= v_{x,k} + \Delta t \cdot a_{TOT,k} \cdot \cos(\gamma_k) \\ v_{z,k+1} &= v_{z,k} + \Delta t \cdot a_{TOT,k} \cdot \sin(\gamma_k) \\ a_{TOT,k+1} &= a_{T,k} + \Delta t \cdot \frac{a_{TOT,k}^2}{g \cdot S_I} . \end{aligned}$$

Next, follow the formulas for the extended Kalman filter given in Anderson and Moore[2] in section 8.2. The time update equations are

$$\left.\begin{aligned} \widehat{x}_{k+1|k} &= \widehat{x}_{k|k} + \Delta t \cdot \widehat{v}_{x,k|k} \\ \widehat{z}_{k+1|k} &= \widehat{z}_{k|k} + \Delta t \cdot \widehat{v}_{z,k|k} \\ \widehat{v}_{x,k+1|k} &= \widehat{v}_{x,k|k} + \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \cos(\gamma_{k|k}) \\ \widehat{v}_{z,k+1|k} &= \widehat{v}_{z,k|k} + \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \sin(\gamma_{k|k}) \\ \widehat{a}_{TOT,k+1|k} &= \widehat{a}_{TOT,k|k} + \Delta t \cdot \frac{\widehat{a}_{TOT,k|k}^2}{g \cdot S_I} \end{aligned}\right\}$$
$$= f_{2D}(X_{k|k}) .$$

The linear expansion for the third, forth and fifth components of f_{2D} about $\widehat{X}_{k|k}$, is respectively

$$\left.\frac{\partial f_{2D}^{(3)}}{\partial v_x}\right|_{X=\widehat{X}_{k|k}} = 1 + \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \left(\frac{1}{\sqrt{\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2}} - \frac{\widehat{v}_{x,k|k}^2}{(\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2)^{3/2}} \right)$$

$$\left.\frac{\partial f_{2D}^{(3)}}{\partial v_z}\right|_{X=\widehat{X}_{k|k}} = \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \left(\frac{-\widehat{v}_{x,k|k} \cdot \widehat{v}_{z,k|k}}{(\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2)^{3/2}} \right)$$

$$\left.\frac{\partial f_{2D}^{(3)}}{\partial a_{TOT}}\right|_{X=\widehat{X}_{k|k}} = \Delta t \cdot \cos(\gamma_{k|k})$$

$$\left.\frac{\partial f_{2D}^{(4)}}{\partial v_x}\right|_{X=\widehat{X}_{k|k}} = \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \left(\frac{-\widehat{v}_{x,k|k} \cdot \widehat{v}_{z,k|k}}{(\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2)^{3/2}} \right)$$

$$\left.\frac{\partial f_{2D}^{(4)}}{\partial v_z}\right|_{X=\widehat{X}_{k|k}} = 1 + \Delta t \cdot \widehat{a}_{TOT,k|k} \cdot \left(\frac{1}{\sqrt{\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2}} - \frac{\widehat{v}_{z,k|k}^2}{(\widehat{v}_{x,k|k}^2 + \widehat{v}_{z,k|k}^2)^{3/2}} \right)$$

$$\left.\frac{\partial f_{2D}^{(4)}}{\partial a_{TOT}}\right|_{X=\widehat{X}_{k|k}} = \Delta t \cdot \sin(\gamma_{k|k})$$

and

$$\left.\frac{\partial f_{2D}^{(5)}}{\partial a_{TOT}}\right|_{X=\widehat{X}_{k|k}} = 1 + 2 \cdot \Delta t \left(\frac{\widehat{a}_{TOT,k|k}}{g \cdot S_I} \right) .$$

In this case the 5×5 state transition matrix, F_k, defined by row where

row one is

$$(1, 0, \Delta t, 0, 0) ,$$

row two is

$$(0, 1, 0, \Delta t, 0) ,$$

row three is

$$(0, 0, \left.\frac{\partial f_{2D}^{(3)}}{\partial v_x}\right|_{X=\widehat{X}_{k|k}}, \left.\frac{\partial f_{2D}^{(3)}}{\partial v_z}\right|_{X=\widehat{X}_{k|k}},$$

$$\Delta t \cdot \cos(\gamma_{k|k})) ,$$

row four is

$$(0, 0, \left.\frac{\partial f_{2D}^{(4)}}{\partial v_x}\right|_{X=\widehat{X}_{k|k}}, \left.\frac{\partial f_{2D}^{(4)}}{\partial v_z}\right|_{X=\widehat{X}_{k|k}},$$

$$\Delta t \cdot \sin(\gamma_{k|k}))$$

and row five is

$$(0, 0, 0, 0, \left.\frac{\partial f_{2D}^{(5)}}{\partial a_{TOT}}\right|_{X=\widehat{X}_{k|k}}) .$$

The measurement matrix is

$$H'_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} .$$

III. Numerical Examples

A few numerical examples are presented to demonstrate the performance of the Kalman filter and to compare it to an $\alpha - \beta$ filter. The examples are all for the one-dimensional case, which proves adequate for filter performance assessment. The displacement dimension is slant range. Velocity and acceleration are total velocity and acceleration. The following details concerning the $\alpha - \beta$ filter may be found in Gray-Murray[3]. The $\alpha - \beta$ filter equations are

$$\begin{bmatrix} \widehat{x}_{k+1|k+1} \\ \widehat{v}_{k+1+k+1} \end{bmatrix} = \begin{bmatrix} (1-\alpha) & (1-\alpha)/T \\ -\beta/T & 1-\beta \end{bmatrix} \begin{bmatrix} \widehat{x}_{k|k} \\ \widehat{v}_{k|k} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} \cdot z_{k+1} .$$

(Since everything in this section is discrete time with no connection to continuous time, it will be convenient to use T as the time variable instead of Δt.)

α and β are related as follows

$$\beta = 2 \cdot (2 - \alpha) - 4 \cdot \sqrt{(1 - \alpha)} \qquad (15)$$

and

$$\beta^2/(1 - \alpha) = (T^4 \sigma_a^2)/\sigma_r^2 . \qquad (16)$$

For comparison and to calculate α and β, let the one σ radar accuracy be 2 mrad and the range from the radar to the target be 250 km. This results in $\sigma_r = 500$ meters. TBM acceleration ranges from about $1g$ before launch to a maximum of about $8g$ at burnout, see Figure 1(a). Hence, $\sigma_a = 80$ m/sec^2. The computed α and β for various values of T (must satisfy both (15) and (16))

TABLE 1.

T (sec)	α	β
0.25	0.132	0.009
1.0	0.431	0.121
2.0	0.673	0.366

For a larger value of T, the $\alpha - \beta$ filter weighs the most recent data more heavily.

Four data sets (numbered **0**-**3**) will be processed by the Kalman filter and the $\alpha - \beta$ filter. Data set **0** is a constant thrust trajectory and data sets **1** - **3** are simulated TBM trajectories. The total acceleration profiles from launch to burnout are given in Figure 1(a).

Next, compute the noise intensities for the constant thrust filter. The measurement noise is the same as in the case of the $\alpha - \beta$ filter, i.e. $\sigma_r = 500$ meters. From (14), the process noise intensity is

$$q = \left(\frac{T \cdot \widehat{a}_{MAX}^2)}{g \cdot \overline{S}_I^2} \right) \cdot \Delta S_I . \qquad (17)$$

From Figures 1(b) and 1(c), $\overline{S}_I = 160$ sec, $\Delta S_I = 40$ sec and $\overline{S}_W = -120$ sec, and as before, $\widehat{a}_{MAX} =$

80 m/sec^2. Using these values in (17), the process noise intensities for various values of T are given in Table 2.

TABLE 2.

T(sec)	0.25	1.0	2.0
q(m/sec^2)	0.24	0.98	1.96

The errors at burn out for data set **0** given by the $\alpha - \beta$ filter and the Kalman filter, assuming varying data measurement rates and no measurement noise, are given in Table 3. ("ERR" denotes measured position minus estimated position, or residual.)

TABLE 3.

T(sec)	$\alpha - \beta$ ERR (m)	K.F. ERR (m)
0.25	389	0.53
1	315	1.55
2	237	2.08

The error for the $\alpha - \beta$ filter decreases with increasing T. It would be expected that the opposite would be the case since more data should give a better estimate. However, it is believed that $\alpha - \beta$ filter behaves in this fashion because of the TBM's hyperbolically increasing acceleration and the fact that as T increases the filter weights more recent data more heavily, i.e. α increases in magnitude. This situation in all likelihood would not happen in the presence of measurement noise. For example, when using $\alpha = 0.132$ on the $T = 1$ data, the resulting error is 4414 meters, and when use $\alpha = 0.431$ on the $T = 2$ data the resulting error is 1100 meters. This deliberate mismatch of α and T simulates data dropout, which in essence is a form of measurement noise. Another reason may be the inaccurate modeling of the system dynamics of the $\alpha - \beta$ filter. Figures 2 through 5 give the results for the 4 data sets. In each of these figures, the solid line represents $T = 0.25$ sec, the dashed line represents $T = 1.0$ sec and the dotted line represents $T = 2.0$ sec. Again, "ERR" means measured position minus estimated position. Figure 6 considers TBM data set **3** as does Figure 5. However, in Figure 6 the filtering starts 51.3 sec after launch, when this TBM is at 10km altitude.

Kalman Filter - Steady State Case

The steady state Kalman filter occurs when the so-called "Kalman gain", (13), is a constant. An appropriate gain may be computed and used even when (13) is not a constant. The resulting filter would be an approximation of the optimal Kalman filter. Needed is a solution to $\Sigma = \Sigma_{k|k-1} = \Sigma_{k|k}$ to (10) and (12). This solution may be computed recursively using (10) and (12). Since acceleration appears in these equations and the actual acceleration varies dramatically during boost, a nominal acceleration, a_{NOM}, must be selected. A different nominal acceleration is used for each measurement rate. Using data set **3**, the error at burn out for the three measurement rates is given in Table 4.

TABLE 4.

T(sec)	0.25	1	2
a_{NOM}	$8 \cdot g$	$8 \cdot g$	$3.2 \cdot g$
ERR (m)	16.7	18.1	18.6

Unique to the case of the two second update rate, the error here actually oscillates between ± 30 meters throughout the entire boost phase. For these three cases, a multi-gain approach using a small number of gains may be developed to give better results without going all the way to the full Kalman filter.

IV. Conclusions

This paper presents a Kalman filter which has potential application to tracking TBMs during boost. The main assumptions of the filter are that the TBM has both a constant thrust and fuel consumption rate during boost. The filter is evaluated and compared to an $\alpha - \beta$ filter by processing data from a hypothetical TBM

which fulfills these assumptions and with a general class of computer generated TBM trajectories representative of the threat set. The results show that while the $\alpha - \beta$ filter does not perform adequately, the constant thrust Kalman filter does. An easy to implement steady state version of the same Kalman filter is presented. This version has adequate albeit poorer performance that the full Kalman implementation.

References

[1] P. Zarchan, *Tactical and Strategic Missile Guidance*, Progress in Astronautics and Aeronautics, Volume 124, AIAA, 1990.

[2] B.D.O. Anderson and J.B. Moore, *Optimal Filtering*, Prentice-Hall, Inc., 1979.

[3] J.E. Gray and W.J. Murray, *Analysis of Alpha-Beta Filters For Potential Applications In Antiair Kill Evaluation Improvements and As Acceleration Detectors*, Naval Surface Warfare Center TR 89-269, Dahlgren, VA 22448, 1989.

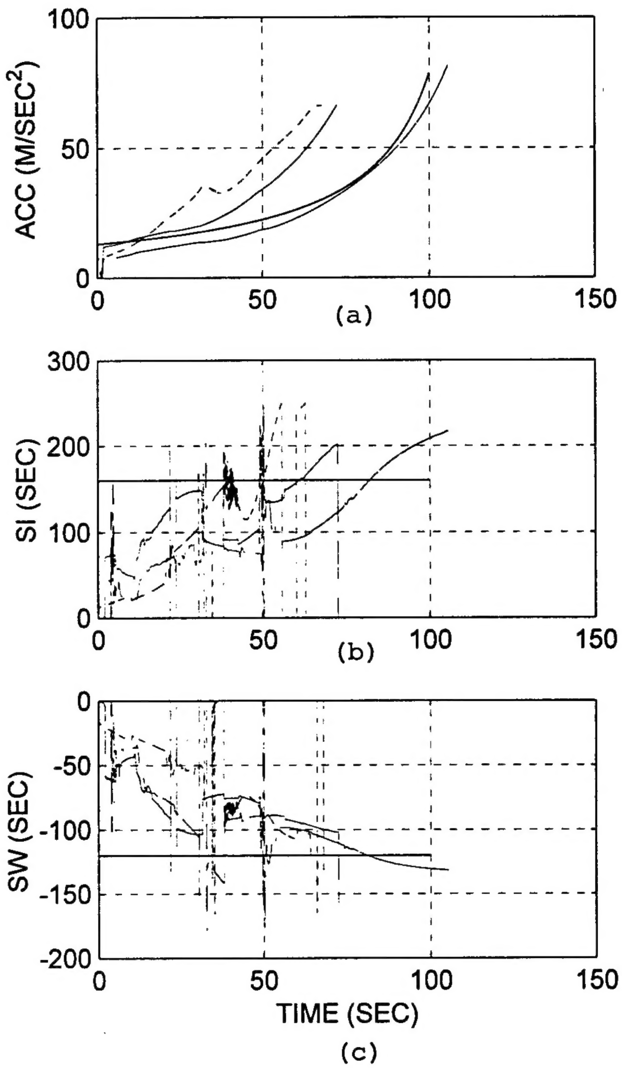

FIGURE 1.

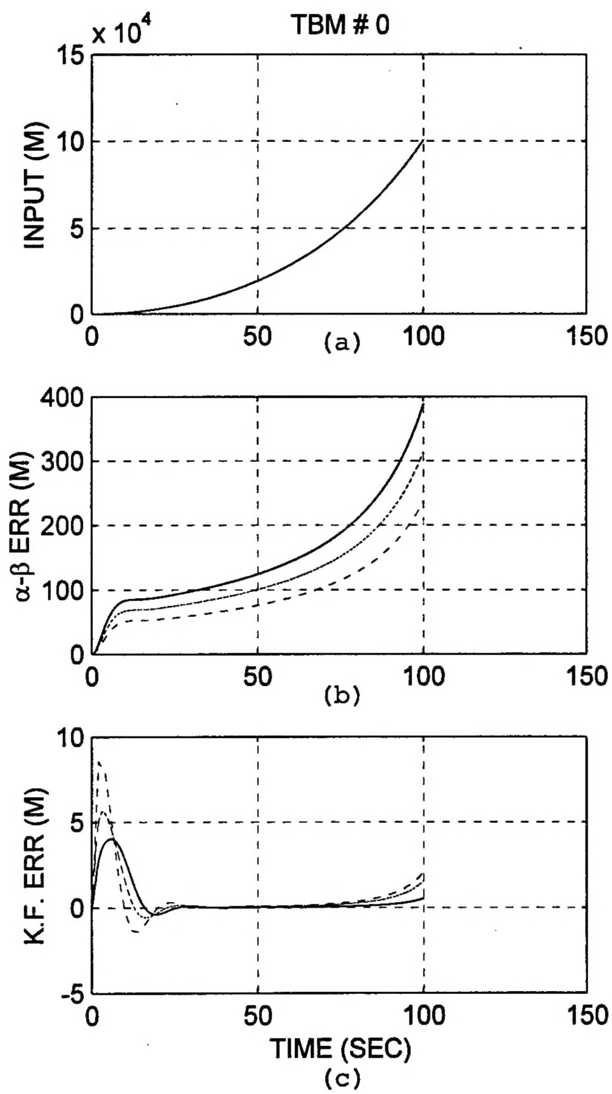

FIGURE 2.

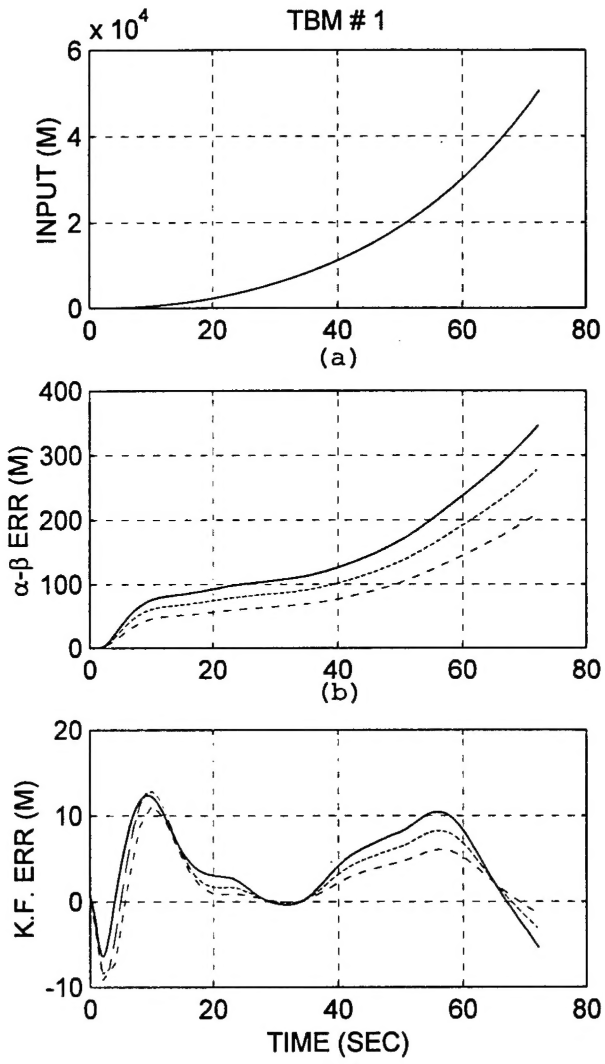

FIGURE 3.

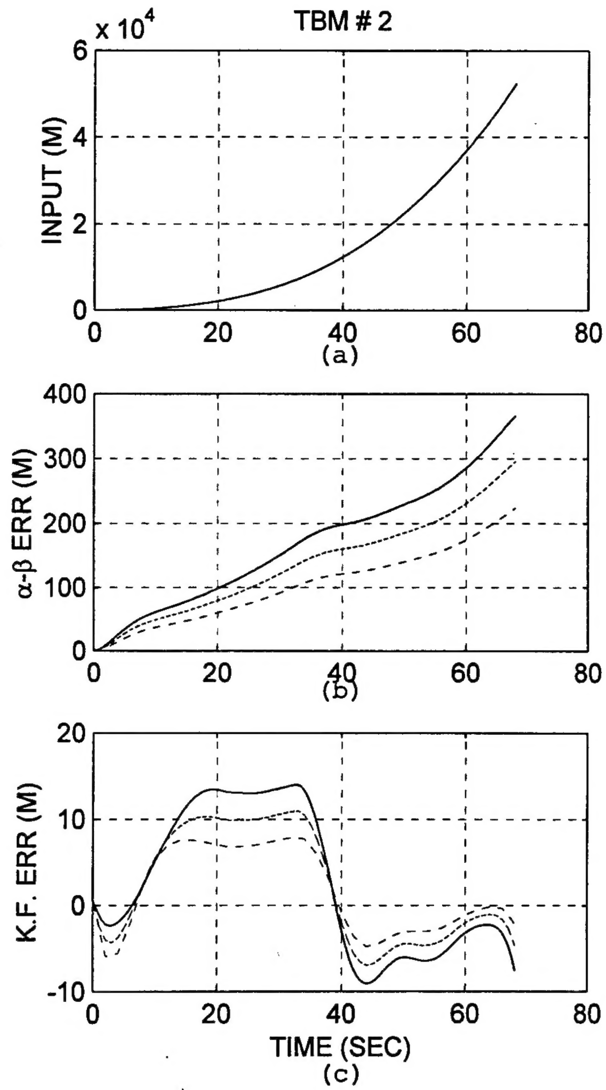

FIGURE 4.

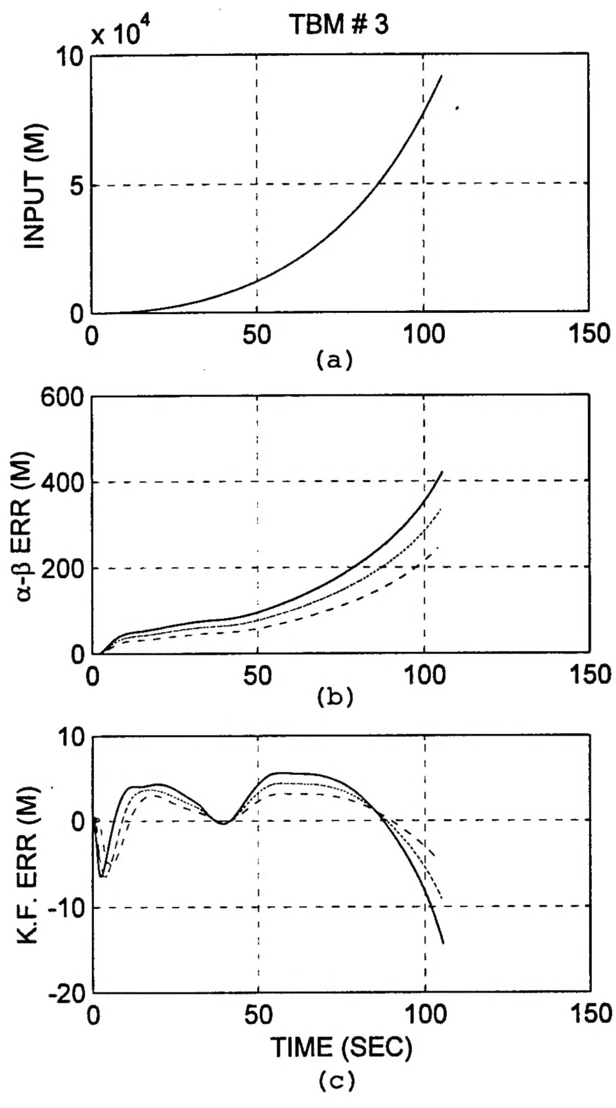

FIGURE 5.

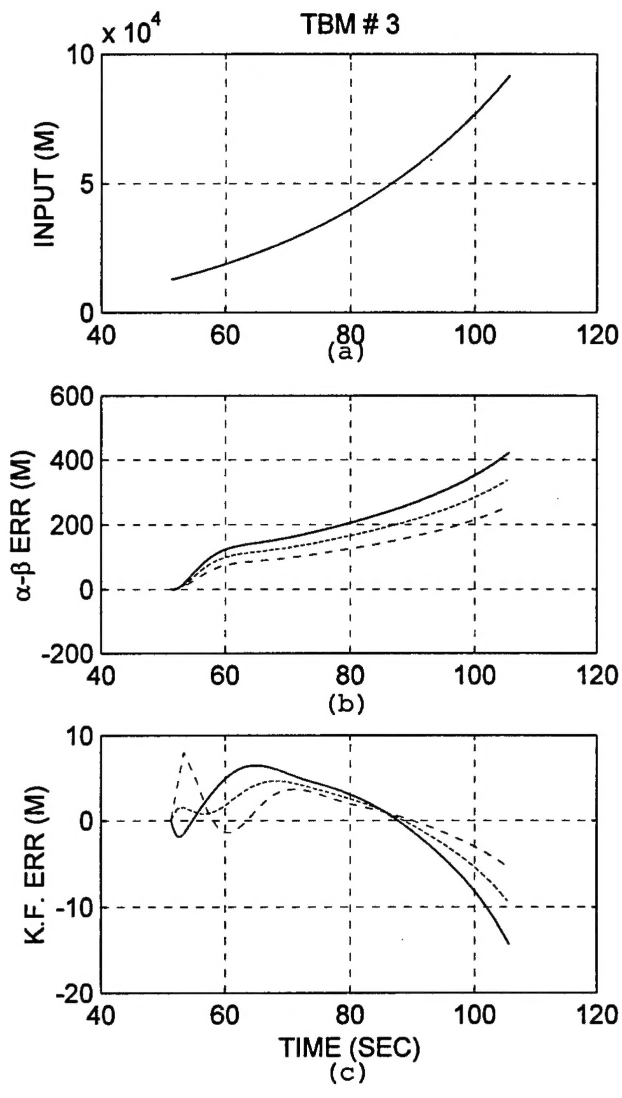

FIGURE 6.